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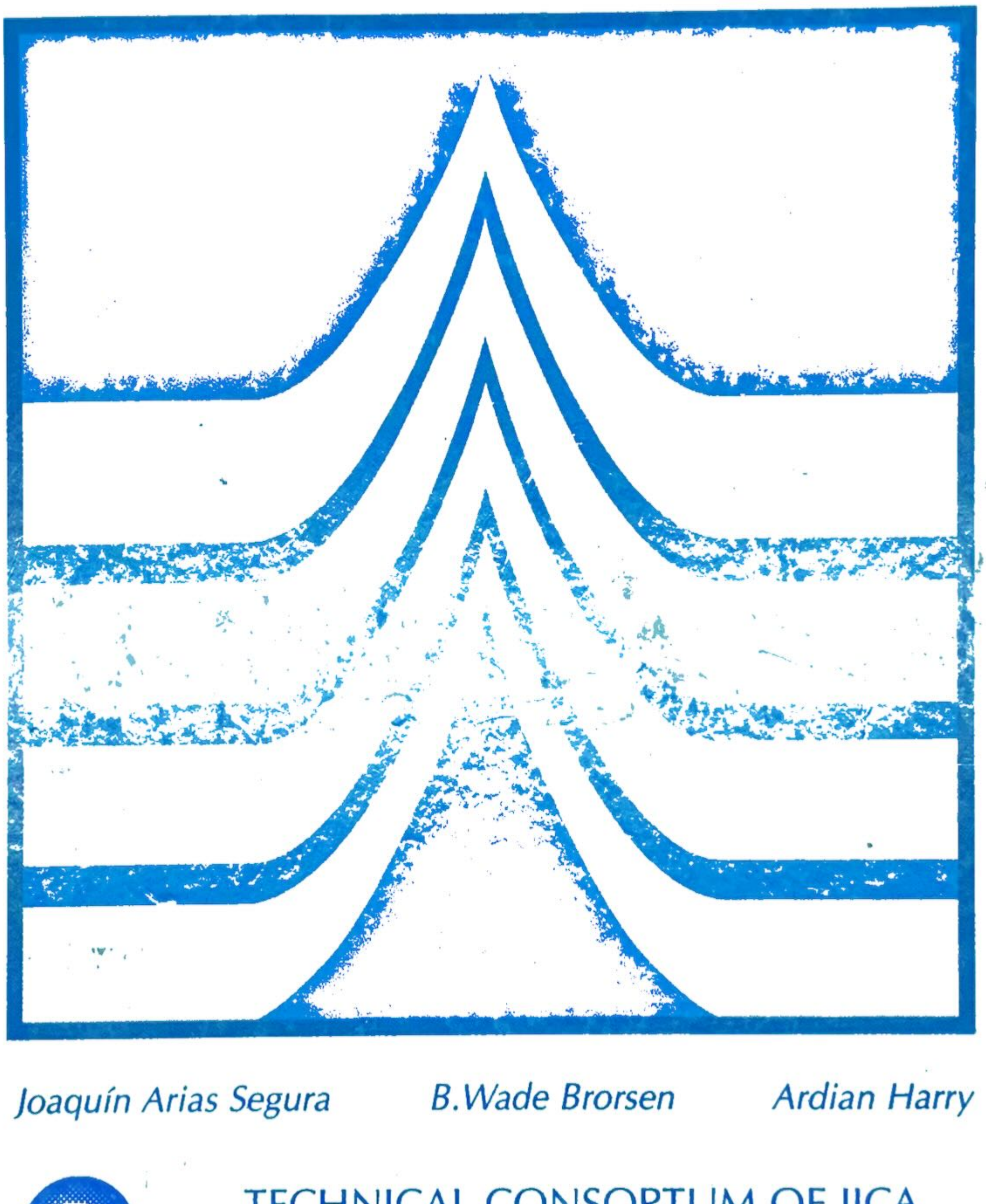
# OPTIMAL HEDGING UNDER NONLINEAR BORROWING COST, PROGRESSIVE TAX RATES, AND LIQUIDITY CONSTRAINTS



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TECHNICAL CONSORTUM OF IICA
AREA OF POLICIES AND TRADE

# Optimal Hedging under Nonlinear Borrowing Cost, Progressive Tax Rates, and Liquidity Constraints

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The views expressed in this book are those of the authors and do not necessarily reflect those of IICA.



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# PRESENTATION

This document deals with the most important factors that affect the decisions of hedging operations in agriculture.

The principal conclusion of this study is that, while producers engage in few hedging operations, this is not necessarily because they are unaware of the benefits of participating in futures markets, but rather because there are certain factors that encourage, and others that discourage, making such a decision. This factors are different for each agricultural activity and agricultural producer.

This topic is important since, in an atmosphere of greater market opening, producers need new marketing instruments that will enable them to minimize risks and increase profits.

A previous version of this document was presented at the Sixth Meeting of the Pan American Association of Commodity Exchanges, as a contribution from the Area of Policies and Trade of the Inter-American Institute for Cooperation on Agriculture (IICA), with a view to encouraging commodity exchanges, brokers and economists and extension agents not to treat all producers the same, or give them blanket recommendations for making hedging operations. The original version (in English) of this document was published in the Journal of Futures Markets (April 2000, volume 20, number 4). The document was translated with the journal's permission.

## ABSTRACT

Empirical research using optimal hedge ratios usually suggests that producers should hedge much more than they do. In this study, a new theoretical model of hedging is derived. Optimal hedge and leverage ratios and their relationship with yield risk, price variability, basis risk, taxes, and financial risk are determined using alternative assumptions. The motivation to hedge is provided by progressive tax rates and cost of bankruptcy. An empirical example for a wheat and stocker-steer producer is provided. Results show that there are many factors, often assumed away in the literature, that make farmers hedge little or none. Progressive tax rates provide an incentive for farmers to hedge in order to reduce their tax liabilities and increase their after-tax income. Farmers will hedge when the cost of hedging is less than the benefits of hedging which come from reducing tax liabilities, liquidity costs, or bankruptcy costs. When tax loss carry back is allowed, hedging decreases as the amount of tax loss carried back increases. Higher profitability makes benefits from futures trading negligible and hedging unattractive, since farmers move to higher income brackets with near constant marginal tax rates. Increasing basis risk or yield risk also reduce the incentive to hedge.



*Key words*: cattle, futures, hedge ratios, markets, nonlinear programming, wheat.

# INTRODUCTION

Empirical research has traditionally defined optimal hedge ratios as the hedge ratio that minimizes price risk. This approach usually finds optimal hedge ratios close to one (Ederington; Howard and D'Antonio; Kolb and Okunev; Mathews and Holthausen; Peck). Lapan and Moschini added basis and yield risk and found lower, but still high, optimal hedge ratios. The reality is that primary producers hedge much less. The theoretical and empirical models used in past research have made simplifying assumptions that restrict them from explaining what farmers actually do. In a sample of 539 Kansas farmers, Schroeder and Goodwin found that, depending upon the crop, only 2% to 10% of the producers raising crops hedged. This suggests that the traditional optimal hedge ratio models are inadequate and therefore it seems worthwhile to explore alternatives.

Tomek argues the hedge ratio is overestimated due to omitting important costs from the farmers' objective function (i.e. yield risk and transaction costs). Lence finds that when some assumptions of the traditional model are relaxed, the optimal hedging strategy is simply not to hedge. Shapiro and Brorsen found that next to income stability (i.e., low income variability), the most important factor explaining the use of futures markets is the individual's debt position. The individual's risk preferences were not important. Thus, an appropriate model might need to have the optimal level of hedging as a function of the farmer's debt position rather than the farmer's risk preferences. The model should explicitly distinguish between a low-leveraged farmer with little financial risk, who may have no need to hedge, and a high-leveraged farmer who might hedge more because of higher expected bankruptcy and liquidity costs. The purpose of this study is to derive a new theoretical model of hedging, considering hedging as a financial action. The financial factors included in the model are progressive tax rates, nonlinear borrowing costs, and liquidity constraints.

Turvey suggests two possible explanations of why hedging increases with debt. The first is that hedging decreases financial risk and the second is that hedging reduces liquidity risk. Several researchers have developed models that capture the trade off between the tax advantage of debt and the debt-related costs (i.e., interests, cost of bankruptcy) (Kraus and Litzenberger; Kim; Bradley, Jarrell, and Kim; Hirshleifer). In these models, it is argued that if the firm can pay its current liabilities, financial leverage decreases the firm's income tax liability and increases its after-tax operating earnings. Smith and Stulz incorporate hedging and conclude that: (i) hedging reduces the variability of pre-tax firm values, which in turn reduces the expected tax liability, increasing the expected post-tax value of the firm; and (ii) hedging benefits shareholders by reducing the expected transaction costs of bankruptcy and increasing the expected after-tax firm value net of bankruptcy costs. These models, though, do not incorporate progressive tax rates, like the U.S. tax code. Several authors (Brorsen; Lence; Turvey; Turvey and Baker) have found that hedging increases with leverage. However, these models do not incorporate any tax structure. In Brorsen's model, interest rates are a nonlinear function of initial wealth, debt, and the variability of ending wealth. However, the ability of hedging to reduce bankruptcy costs, liquidity costs and tax liabilities are not incorporated in Brorsen's model.

Collins provides an excellent review of the work related to optimal hedge modeling and also derives a positive model of hedging. The appeal of Collins' model is that it is simple and easily understood. Collins' model can be viewed as a special case of the model derived here. Collins' model uses a linear borrowing cost function and does not account for taxes. Also, the safety constraint in Collins' model could be made equivalent to the liquidity cost imposed in this paper. A value of the probability that end-of-period equity is less than some disaster level ($\alpha$ in Collins' model), can be calculated for the equivalent liquidity cost in our model. Collins forces the probability of bankruptcy to be zero. We can do the same thing by letting the cost of bankruptcy be infinite. The new theoretical model developed here incorporates a nonlinear interest rate function, and the function is estimated by determining the expected bankruptcy losses to debt holders. Liquidity is the motivation for farmers' use of futures in Turvey, and Turvey and Baker's model. With variable yields and prices, the ability of farms to generate sufficient cash to meet financial commitments is not certain (Turvey and Baker). The cash flow derived from hedging is the difference between the net price received with the hedge and the cash price which would have been received without the hedge. High-debt farms are more likely to have liquidity constraints and would hedge more. Turvey and Turvey and Baker include liquidity cost when firms have to sell off long-term assets to cover losses. The new theoretical model explicitly allows hedging to reduce bankruptcy and liquidity costs and tax liabilities.

The minimum variance model of Johnson and Stein has been used widely to study hedging behavior. There is no reason to believe that utility will be maximized when the variance of the spot minus futures position is minimized (Benninga, Eldor and Zilcha). Benninga, Eldor, and Zilcha, cited by Lence, show sufficient conditions under which the minimum-variance hedge would be consistent with expected utility-maximizing hedge ratios. The basic assumptions are that (i) the decision maker is not allowed to borrow, lend, or invest in alternative activities, (ii) there are neither safety margins nor futures brokerage fees, (iii) production is deterministic, (iv) random cash prices can be expressed as a linear function of futures prices plus an independent error term, and (v) futures prices are unbiased. All assumptions, except assumption (v) are relaxed in this study.

The model in this study is derived under the assumption that farmers have risk-neutral preferences. Firms are normally assumed risk averse. However, empirical evidence shows that risk preferences are not significantly related to hedging (Shapiro and

Brorsen). Schroeder and Goodwin found that risk preferences of crop producers did not influence forward pricing. Williams, Smith and Stulz, and Brorsen show that risk aversion is not necessary for firms to hedge. Rather than assuming risk aversion, this study assumes that firms maximize expected equity. The objective function is concave for reasons other than risk aversion. Specifically, progressive tax rates and a nonlinear interest function cause the objective function to be concave. The model allows interest rates to vary according to the probability of bankruptcy, accounts for the trade off between the tax-reducing benefits of hedging and the cost of hedging, and also allows hedging to be a source of meeting cash flow requirements. Progressive tax rates provide the incentive to hedge when the firm is not close to bankruptcy. Farmers reduce the variability of their taxable income and as a result their taxes and at the same time increase their after-tax income by hedging when facing progressive tax rates.

It has been widely accepted that output and price risk should be considered together when estimating optimal hedge ratios. Yet few studies have considered both. Exceptions are Chavas and Pope, Grant, Lapan and Moschini, Losq, and Rolfo. Lence and Tomek argue that one of the most important restrictions of the minimum-variance hedge ratios seems to be that production is deterministic (the other important restriction is no alternative investments). Little hedging can be partly explained by farmers being unable to adequately forecast the size of the harvest even after all production deci-sions have been made (Rolfo). Production risk provides an additional explanation of optimal hedge ratios being below one (Rolfo; Chavas and Pope; Lapan and Moschini; Losq). Lence concludes that stochastic production reduces significantly both optim hedges and the opportunity cost of not hedging. Also, the optimal hedge is a decreasing function of the variance of the production disturbance (Lapan and Moschini; Lence).

In this study optimal hedge ratios for a wheat and stocker-steer producer and their relationship with yield risk, price variability, basis risk, and financial risk are determined using a new hedging model. Because of the complexity of the analytical model, comparative statics cannot be derived analytically. Therefore, the effects of various factors are determined numerically for a specific example. The factors considered are: variance of both cash and futures prices, basis risk, yield variance, progressive tax rates, cost of hedging, liquidity cost, off-farm income, deterministic production, and tax-loss carry back. The objective function could not be integrated analytically and, therefore, Monte Carlo integration is used.

# THE MODEL

Assume that because of unpredictable conditions (i.e., weather variability), the decision maker cannot predict output with certainty at the time of the production decision. The technology of the firm can then be represented by the stochastic production function:

(1) $$y_t = f(K_{t-1}, X_{t-1}, \varepsilon_{1t})$$

where yt is an n-dimensional vector of production levels at time t (letting $t$ = 1 be the harvest time and $t$ - 1 the time where decisions are made), $f$ is an increasing function of $K_{t-1}$ and $X_{t-1}$, $X_{t-1}$ is a vector of inputs, $K_{t-1}$ is capital assets, and $\in_{1t}$ is an $n$x1 vector of error terms.

Assume a competitive producer whose only available hedging instrument is a futures contract. Let $F_t$ be an $n$-dimensional vector of the amounts hedged in the futures market. Firms are not allowed to take a long position in the futures market. For each unit of $F_t$, the firm makes $(P^f_{t-1} - P^f_t)$, the difference between the selling and the buying prices of the futures contracts. In the cash market, the firm makes $(P^f_t + b_t)$ dollars for each unit of output sold, where the basis ($b_t$) is the difference between the cash price $(P^c_t)$ and the futures price at harvest $(P^f_t)$. The firm's variable costs consist of the cost of inputs $(P^{x\prime}_{t-1} X_{t-1})$, the cost of hedging $(h^{\prime}_{t-1} F_t)$, interest on debt ($i_t D_t$), and depreciation ($\alpha K_{t1}$), where $P^x_{t-1}$ is a vector of input prices, $h_t$ is the cost of hedging, $i_t$ is the interest rate, $D_t$ is total liabilities, and $\alpha$ is a (constant) rate of depreciation. The firm's profit at time $t$ ($\pi_t$) can then be represented by:

(2) $$\pi_t = y^{\prime}_t(P^f_t + b_t) + F^{\prime}_t(P^f_{t-1}\ P^f_t) - P^{x\prime}_{t-1} X_{t-1} - h^{\prime}_{t-1} F_t - i_t D_{t-1} - \alpha K_{t-1} - \gamma L_t$$

A liquidity fee ($\gamma L_t$) has been included in (2) to account for situations when the firm cannot honor its current obligations; where $\gamma$ is a liquidity fee rate, and $L_t$ is the amount of current liabilities that the firm could not pay when liquid capital was not available. As assumed by Turvey and Baker, firms incur liquidity costs when they must sell off long-term assets to cover losses. Taxable income ($TI_t$) can be obtained by subtracting the corresponding standard deductions ($STD$) and exemptions ($EXM$) from (2) ($TI_t = \pi_t$ - STD - EXM). Note that because all payments to debt claims are assumed to be tax deductible, interest is subtracted in (2). Total liabilities ($D_t$) subject to interest charges can be determined by computing the liabilities from the previous period, plus the capital needed to finance inputs, hedging costs (margin calls are also included here), and investment, minus current assets from period $t$ -1 ($CA_{t-1}$), minus any payment made to the principal in period $t$ -1 ($A_{t-1}$).

(3) $$D_t = D_{t-1} + P^{x/}_{t-1} X_{t-1} + h^{/}_{t-1} F_t + (K_t - K_{t-1}) - CA_{t-1} - A_{t-1}$$

where ($Kt - K_{t-1}$) is capital investment net of depreciation.

Denote $\tau(TI_t)$ as the firm's income tax rate, which is an increasing function of taxable income ($TI_t$); therefore, net income ($NI_t$) or after tax income is indicating that taxes are zero whenever taxable income is negative, and net income can be negative whenever profits are negative, but its absolute value cannot be less than beginning equity[2], $W_{t-1}$ (i.e., the firm cannot lose more capital than it has available).

(4) $$NI_t = \begin{cases} [1 - \tau(TI_t)]\, TI_t; & TI_t > 0 \\ \pi_t; & TI_t < 0 \text{ and } \pi_t > -W_{t-1} \\ -W_{t-1}; & \pi_t < -W_{t-1} \end{cases}$$

1 Losses beyond $-W_{t-1}$ will be incurred by the farmer's lending institution.

The decision maker knows at time $t$-1 the vector of futures prices ($\mathbf{P}^f_{t-1}$), the price vector of inputs ($\mathbf{P}^x_{t-1}$) and the cost of selling in the futures market ($h_t$). However, because of production lags the farmer does not know with certainty the vector of output prices in the cash market ($P^c_t$), or the futures price at harvest ($\mathbf{P}^f_t$). The random variables, ($P^f_t$) and ($P^c_t$), follow a random walk with the corresponding correlated vectors of mean zero errors, $\varepsilon_{2t}$ and $\varepsilon_{3t}$.

The firm's net worth would be net income plus beginning equity: $W_t = NI_t + W_{t-1}$. Since the firm cannot lose more capital than it has available, ending wealth ($W_t$) cannot be negative and net income is bounded to be greater than the negative of beginning equity ($NI_t \geq W_{t1}$). Note that the firm will be bankrupt when profits are negative and less than the negative of initial wealth ($\pi_t < W_{t-1}$). When ($W_t = 0$) and the amount ($W_{t-1} - \pi_t$) is positive, the firm is bankrupt and the losses to debt holders ($LS_t$) are:

(5) $$LS_t = C(D_t) + W_{t-1} - \pi_t\,;\ \pi_t < -W_{t-1}$$

where $C(Dt)$, the bankruptcy fee, is an increasing function of the debt level (3).

Farmers face risk (i.e., yield risk, basis risk, price risk); therefore, bankruptcy is possible since the condition ($\pi_t < -W_{t-1}$) (4) can occur with positive probability. Assuming banks are risk neutral, lenders will charge the firm a premium which would be a function of the expected bankruptcy losses. Hence a firm with a higher probability of bankruptcy would have a higher expected rate of interest on its debt than a firm with lower financial risk (Barry, Baker, and Sanint). In practice, interest rates can vary by as much as five percentage points (Turvey and Baker). It is assumed here that banks have enough information[3] about the firm to calculate the expected bankruptcy losses and charge a premium to firms likely to go bankrupt. The firm's expected interest rate on debt ($it$) will then equal the prime interest rate ($r^*$) plus a premium (Prt): $it = r^* + PRt$ .

Given the bankruptcy losses ($LS_t$) in (5), the premium ($PR_t$) can be defined as the expected ratio of the losses to total liabilities ($LS_t/D_t$):

(6) $$PR_t = \iiint I[\pi_t < -W_{t-1}] \frac{LS_t}{D_t} f(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t})\, d\varepsilon_{1t}\, d\varepsilon_{2t}\, d\varepsilon_{3t}$$

where I[.] is an indicator function that takes the value of one if the firm goes bankrupt and zero otherwise. $f(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t})$ is the probability density function of the error terms for yields, and the cash and futures prices respectively. The premium ($PR_t$) at time t would be zero if the probability of the firm going bankrupt is zero so that the ratio of losses ($LS_t$) to the debt level ($D_t$) equals zero.

Assume a one-period model and that the hedger aims to maximize the expected net present worth ($W_1$), the objective function is:

(7) $$\max_{F_0} E[W_1] = W_0 + \iiint \beta\, NI_1\, I[\pi_1 > -W_0]\, f(\varepsilon_{11}, \varepsilon_{21}, \varepsilon_{31})\, d\varepsilon_{11}\, d\varepsilon_{21}\, d\varepsilon_{31}$$

2 In their decision regarding granting a loan to a farmer, banks consider not only financial indicators about the farm but also agricultural indicators like the type of crop and the risk associated with the crop. The local Product credit Association (PCA) uses a logit model to predict the probability of bankruptcy of their borrowers. It is true that the lender's decision is a step function with only two possible outcomes (Yes or No), but the existence of several lending institutions with different interest rates, allows for borrower's interest function to be approximated as being continuous. The logit model used by the PCA does not consider hedging. A bank, however, would likely not want to loan to a grain elevator that had large unhedged positions.

where β is the discount factor, ($0 < \beta < 1$) and $W_0$ is the exogenous initial wealth. In (7), when net worth is negative, ($\pi_1 < -W_0$) $W_1$ will equal zero, indicating that equity holders get nothing if the firm enters bankruptcy. At time 0, the farmer has already decided the input levels ($X_0$), the level of capital ($K_0$) and the level of output. The only choice variable is how much to hedge ($F_0$). The debt level is determined by (3), and it will change as beginning equity changes.

## DATA AND PROCEDURES

Because of the complexity of the analytical model (7), no analytical results were derived. Therefore, the effects of various factors are determined numerically for a specific example. Simulations are performed for a wheat and stocker steer producer. The model is simulated assuming acreage and number of steers are fixed in a single period. Government programs for wheat are not included to simplify the model. The producer has made the decision to produce 1000 acres of wheat, and graze 296 steers on the winter wheat pasture. The wheat is planted in September and harvested in June, and the farmer buys steer calves in October and sells them in March. This study assumes resource constraints and limited choices. By fixing output to a certain level, the only choice variable in the model is how much of the expected output should be hedged. The data for the base model is set forth in table 1. Simulations are first performed by changing initial wealth ($W_0$) and solving for the optimal hedge ratio $F_0$ in (7). In the base model, initial wealth ($W_0$) is set at $150,000 (ending debt to assets ratio is approximately 0.62), and then numerical derivatives were used to obtain the response of the optimal hedge ratio to variations in the parameters of the model.

Equations (6) and (7) cannot be integrated analytically, therefore Monte Carlo integration was used. A total of 5000 random numbers for yields (1) were generated from a beta distribution and the cash and futures prices were generated from a lognormal distribution with a correlation of 0.9 and mean and standard deviations shown in table 1. The beta distributions were generated by

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**Table 1. Wheat for Grain, Owned Harvest Equipment, Budget Per Acre and Stocker Steers on Wheat Pasture Cost/Returns Per Head[a]**

| Variable | Units | Value | Standard Deviation |
|---|---|---|---|
| Wheat | | | |
| Variable Cost | $/acre | 78.32 | - |
| Capital investment | $/acre | 171.47 | - |
| Yield | bu./acre | 35.00 | 5.00[b] |
| Cash price | $/bu. | 2.90 | 0.55 |
| Futures price at planting | $/bu. | 3.20 | - |
| Futures price at harvest | $/bu. | 3.20 | 0.50 |
| Cost of hedging | $/bu. | 0.02c | - |
| Steers | | | |
| Variable Cost | $/head | 70.87 | - |
| Capital investment | $/head | 29.09 | - |
| Steers calf Weight | cwt | 4.36 | - |
| Sale weight | cwt | 6.65 | 0.50[d] |
| Price of steers calves | cwt | 92.00 | - |
| Cash price | $/cwt | 82.00 | 5.50[e] |
| Futures price at planting | $/cwt | 85.00 | - |
| Futures price at harvest | $/cwt | 85.00 | 5.00 |
| Cost of hedging | $/cwt | 0.25 | - |
| Other Variables | | | |
| Bankruptcy fee | % | 30.00 | - |
| Liquidity fee | % | 1.00 | - |
| Interest rate | % | 8.50 | - |
| Depreciation | | 1/7 | - |

a The costs for production were taken from the OSU Enterprise Budgets developed by
Oklahoma State University, Department of Agricultural Economics.

b Source: Schroeder and Goodwin

c Source: Brorsen, Coombs and Anderson

d Source: Koontz and Trapp

e Source: USDA. Calculated as the standard deviation of the cash price changes from October to march, 1980-1991.

first drawing two independent samples from a Gamma distribution using the Phillips generator (Shannon, p.365). These two samples are then used to get beta random numbers (Naylor et al.). Cash and futures prices were generated assuming that futures markets are efficient. Mean and variances were calculated to check if they conform with original numbers. Antithetic variates (Shannon) were used to increase precision for a given sample size. The discounting factor is $\beta = 1/(1+0.085)$. The model was solved to obtain the optimal hedge ratios ($F_0$) by means of the nonlinear algorithm in GAMS/MINOS (Brooke, Kendrick, and Meeraus).

## *Bankruptcy*

Handling bankruptcy requires introducing discrete variables into the model and makes the problem difficult to solve. A dummy variable is needed to calculate the bankruptcy fee. To avoid discontinuities and nondifferentiability, approximation to the 0-1 variable is necessary. Let the bankruptcy dummy variable $Z_t$ equal one when the firm is bankrupt (equity ($W_t = 0$)) and zero otherwise ($W_t > 0$). Define $Z_t$ as: $Z_t = \exp[-M\ W_t]$, where $M$ is a large number, and $W_t$ is positive. Notice that when the firm is bankrupt, $W_t = 0$, and $Z_t = 1$. On the other hand, whenever the firm is not bankrupt $W_t > 0$, the program will be calculating the exponential of a large negative number which makes $Z_t = 0$ in the limit. Therefore; the only possible values for $Z_t$ are one and zero, given that $M$ is large enough and $Wt$ is greater than or equal to zero. Even though equity ($W_t$) is restricted to be positive, GAMS would allow $W_t$ to be negative while reaching the optimal solution. If $W_t$ becomes negative, $Z_t$ would approach infinity and the program will fail to converge. To avoid this problem, $Z_t$ was restricted using the transformation:

(8) $$Z_t = \exp[1 - 0.5(\sqrt{(1 + M\,W_t)^2} + 1 + M\,W_t)]$$

With this transformation, $Z_t$ is not allowed to be greater than exp(1), and the computer will continue solving even when equity goes negative. An alternative to the 0-1 variable would be integer programming, but that would make the program too slow and difficult to solve. Results were checked to confirm that $Z_t$ yielded 0-1 values. Alternate starting values were used to check any problems with convergence to local maximums.

## *Progressive Tax Rates*

Assume the farmer is married with two children. According to the 1994 instructions of tax form 1040 (Internal Revenue Service (IRS)), the standard deductions equal \$9800 (\$2450 for each) and total exemptions equal \$3175. These quantities are subtracted from profits ($\pi_t$) to get taxable income ($TI_t$). To calculate the tax, Schedule y-2 (IRS) for married filing separately was used. The tax schedule is a step function which was made differentiable following the same principle as in (8). At first, inequality constraints were imposed, however the programs became slow and exceeded storage constraints.

## *Tax Carry Back*

If the farmer's income for the year ($\pi_t$), is negative then there is a net operating loss (*NOL*) (IRS, publication 536) equal to: $NOL_t = -(\pi_t)$. The *NOL* can be used by deducting it from income in another year or years. Assume the farmer can carry back his/her tax losses only to the previous year. Assuming the farmer stays in the same income bracket, the tax rate ($\tau(TI_{t-1})$) is the same before and after deducting the *NOL*, then the tax losses ($TXLS_t$) is: $TXLS_t = \tau(TI_{t-1})NOL_t$. This is the amount that should be refunded to the farmer and added to net income ($NI_t$) in (4). Tax loss carry backs do not fully remove the incentive to hedge since the farmer loses the standard deduction when there is a tax loss.

## RESULTS

The base model results (debt to assets ratio of 0.62) show that the optimal hedge ratios for wheat and steers are 0.45 and 0 respectively. Hedge ratios for wheat are higher than those for steers since wheat is relatively riskier given that the time between planting and harvest for wheat production is longer than the time the steers are owned.

### *Debt to Assets Ratio and the Optimal Hedge Ratio*

Decisions at a certain level of income and debt to asset ratio depend on whether the farmer is solvent or not and depend on the potential for bankruptcy. The model yields optimal hedge ratios for wheat in the range of 0.35 to 0.87 as the debt to asset ratio varies from 0.02 to 0.89 (fig. 1). Results indicate that the farmer would choose to hedge steers only when the probability of bankruptcy is positive (figs. 1 and 2). Bankruptcy is an extreme case since it only happens when the debt to asset ratio is higher than 0.8 (fig. 2).

Surprisingly to us, the model shows that hedging decreases with increasing leverage (fig. 1) when the firm has zero probability of going bankrupt (fig. 2). The higher debt decreases income by increasing interest paid and lowers the average marginal tax rate which reduces the incentive to hedge.

Before the firm goes bankrupt, it will first encounter liquidity problems, and hedging should serve as a source of cash flow (Turvey and Baker). This is true as long as the cost of hedging is lower than the liquidity cost. The liquidity cost in figure 1 is 1% of outstanding short-term liabilities. As soon as the firm experiences

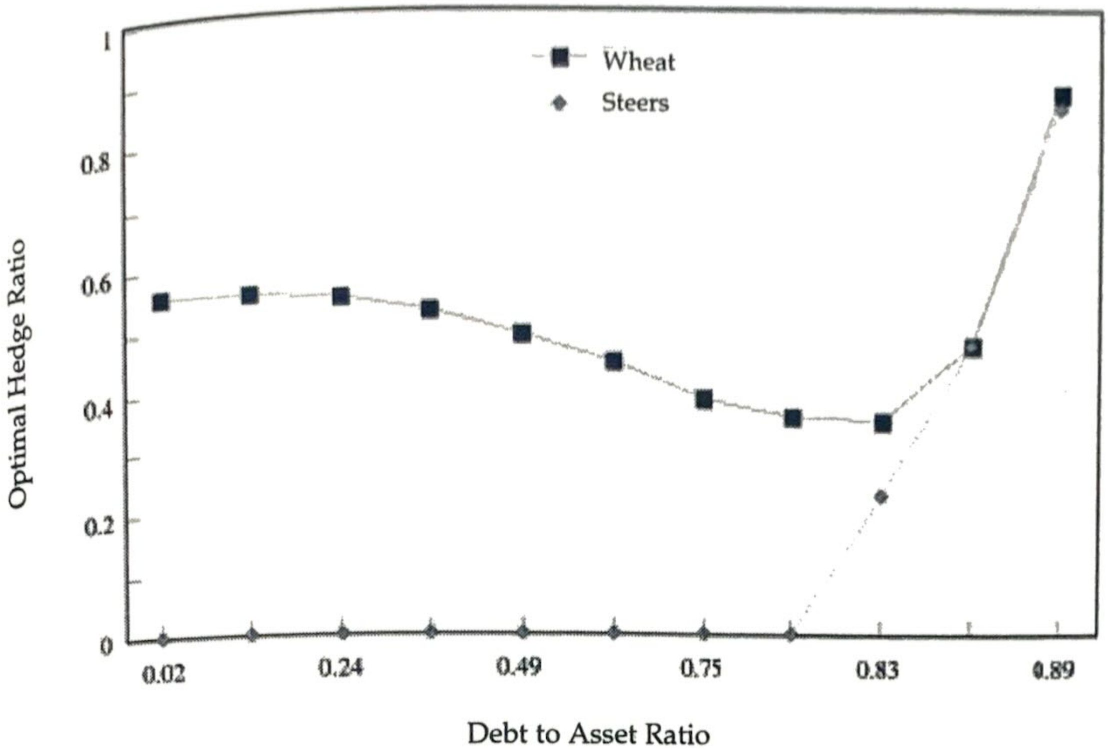


**Figure 1. Optimal hedge ratio vs. debt to asset ratio for a wheat and stocker steer producer.**

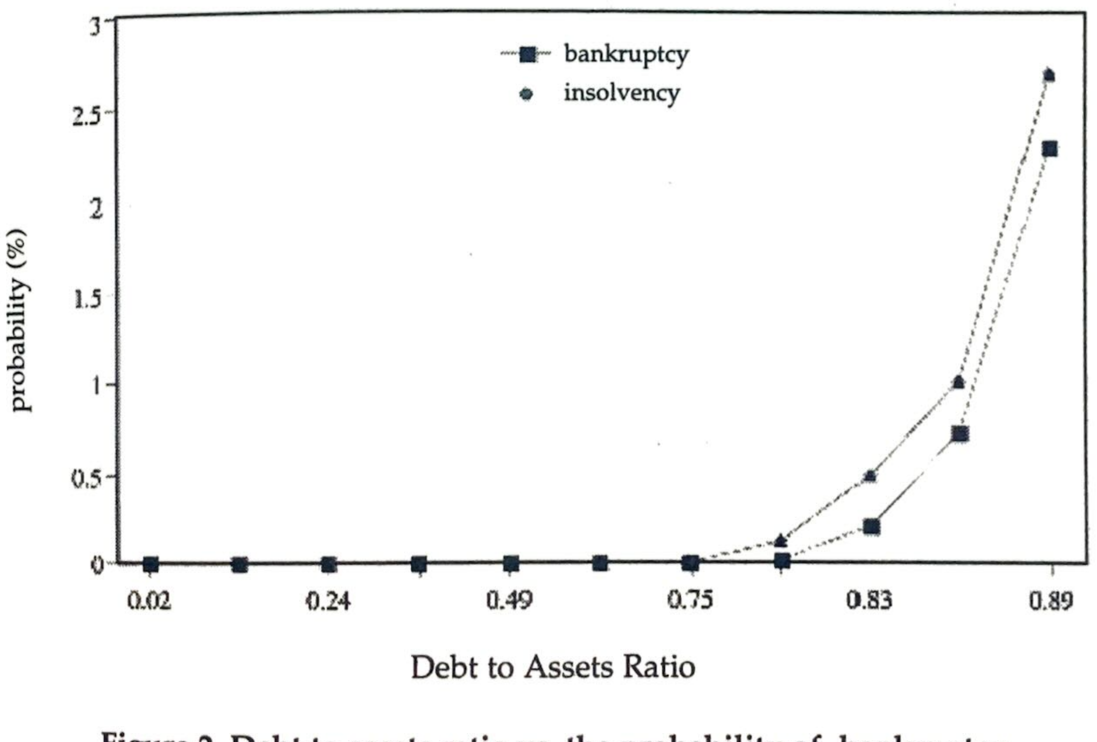


**Figure 2. Debt to assets ratio vs. the probability of bankruptcy and insolvency (in %).**

both insolvency and bankruptcy (fig. 2) the firm is willing to pay the cost of trading more in the futures market (fig. 1) and reduce the relatively higher liquidity and bankruptcy costs. Note that the results hold under the assumption of risk neutrality.

As Collins and Karp argue, leverage increases the probability of a disaster which causes increased risk of loss for the lender, and the cost of borrowing also increases with leverage. This relationship is shown in figure 3. When the probability of bankruptcy is non-zero, the interest rate rises above the riskless interest rate. For this simulation example, the firm shows positive probability of bankruptcy at relatively high debt to assets ratios (above 0.8), where the cost of borrowing begins to rise.

## *Cost of Hedging*

Traditionally optimal hedging models introduced by Johnson and Stein and variations of their approach (Myers and Thompson) assume away the cost of hedging. Here the cost of hedging influences greatly the decision of whether or not to hedge and how much to hedge (figs. 4 and 5). Similar results were obtained by Lence and Berck. The cost of hedging has to be lower than 15 cents/cwt for a cattle producer to hedge. A cattle producer would hedge more than 65% of the steers if the cost of hedging is as low as 5 cents/cwt. The cost of hedging alone might be the reason why a cattle producer would not hedge. For a wheat producer, the optimal hedge ratio varies from a high of 0.65 to 0.14 when the cost of hedging increases respectively from 14 cent/bu to 28 cents/bu.

Clearly, transaction costs of hedging may well exceed the tax-reducing benefits of hedging, causing optimal hedge ratios to decrease. These results apply for a debt to asset ratio of 0.62, a point at which the farmer does not encounter any liquidity or bankruptcy cost.

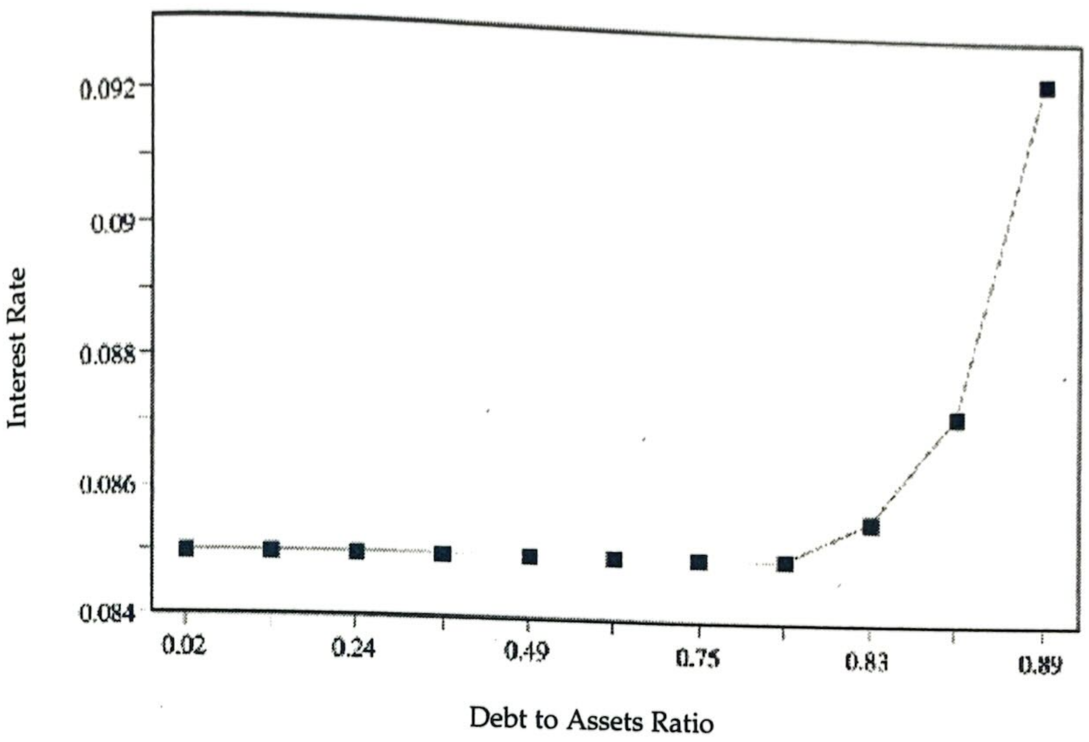


**Figure 3. Debt to assets ratio vs. the interest rate.**

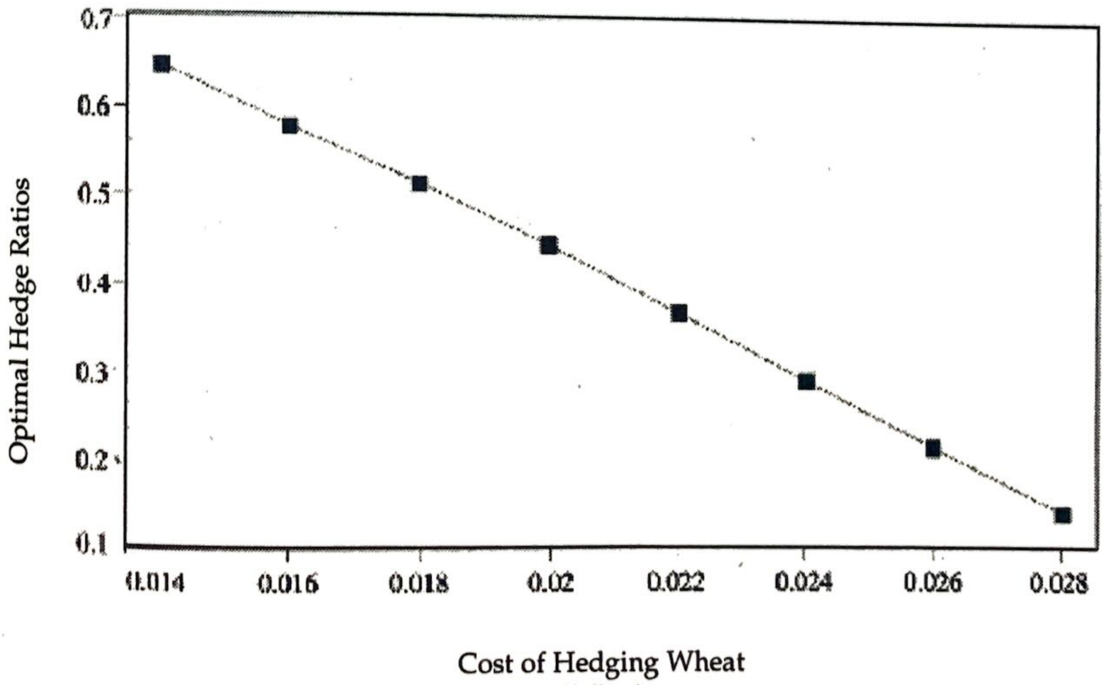


**Figure 4. Optimal hedge ratios vs. the cost of hedging wheat.**

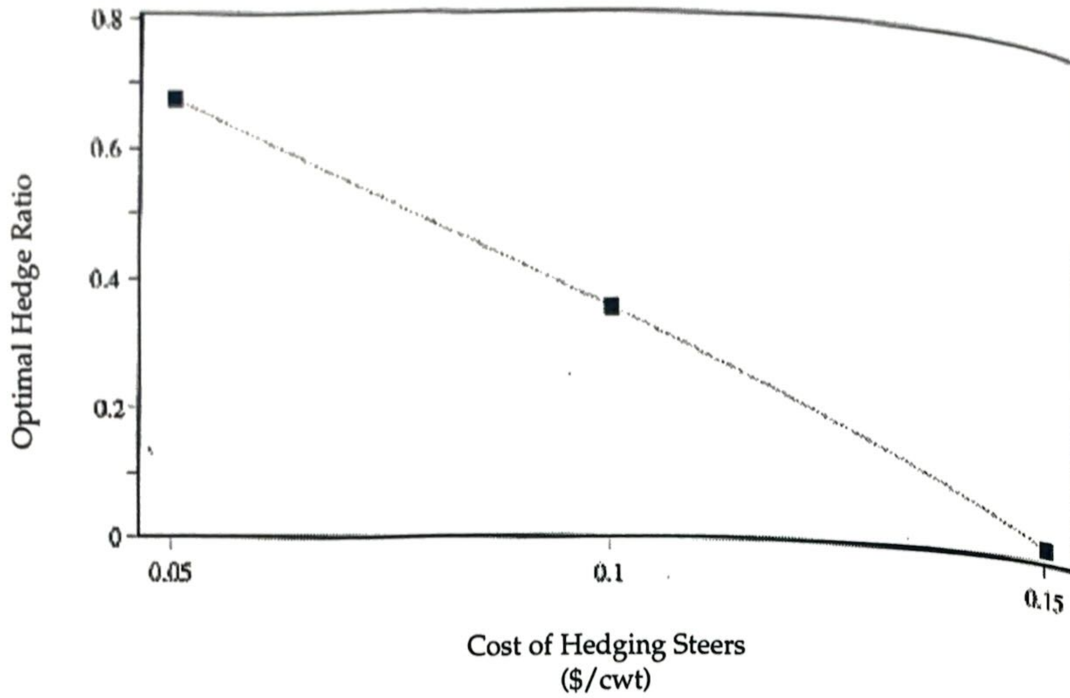


**Figure 5. Optimal hedge ratio vs. the cost of hedging steers**

## *Off-Farm Income*[4]

In agreement with Lence's results, optimal hedge ratios are greatly influenced by off-farm income (fig. 6). With low profitability, the producer pays almost no taxes after deductions and exemptions, which means that the tax reduction with increasing hedging is minimal. As off-farm income increases, the producer pays more taxes and the tax-reducing benefits of hedging become more attractive resulting in the producer hedging more. Hedging reduces the variability of profits and the expected tax liability which in turn increases the expected ending wealth. This result could not be found by Lence since tax effects were ignored in his study. After a level of off-farm income of $20,000 (the average farm income is $5,750), the optimal hedge ratio begins to decline.

4 The effect of higher profitability works much the same way as off-farm income. As the cash position becomes more profitable, benefits from futures trading become relatively negligible, making hedging unattractive. With a 25% increase in prices, profitability increases to an average of $71,908, a point at which wheat hedging is almost zero.

This result may not be due to what Lence calls "the dilution effect." In Lence's work, as the share of the alternative investment to initial wealth increases, the cash position becomes relatively less and less important to the farmer, and the incentive to hedge decreases. In this study, though, as income increases, the farmer moves up to a higher income bracket where marginal tax rates are lower and the incentive to hedge is less. The nonlinearity clearly displayed in figure 6 is most likely due to the tax schedule, which is a step function.

## *Variance of Cash and Futures Prices*

For a wheat producer, the optimal hedge ratio is positively related to the variance of cash prices (fig. 7). As the cash price risk increases, the optimal hedge ratio increases. Under the assumptions of production certainty and basis risk, Robinson and Barry, Peck, and Kahl and others found the same result. In this study, however, we assume the producer is risk neutral and production is uncertain. The zero hedging position for steers did not change with variations in the variance of prices.

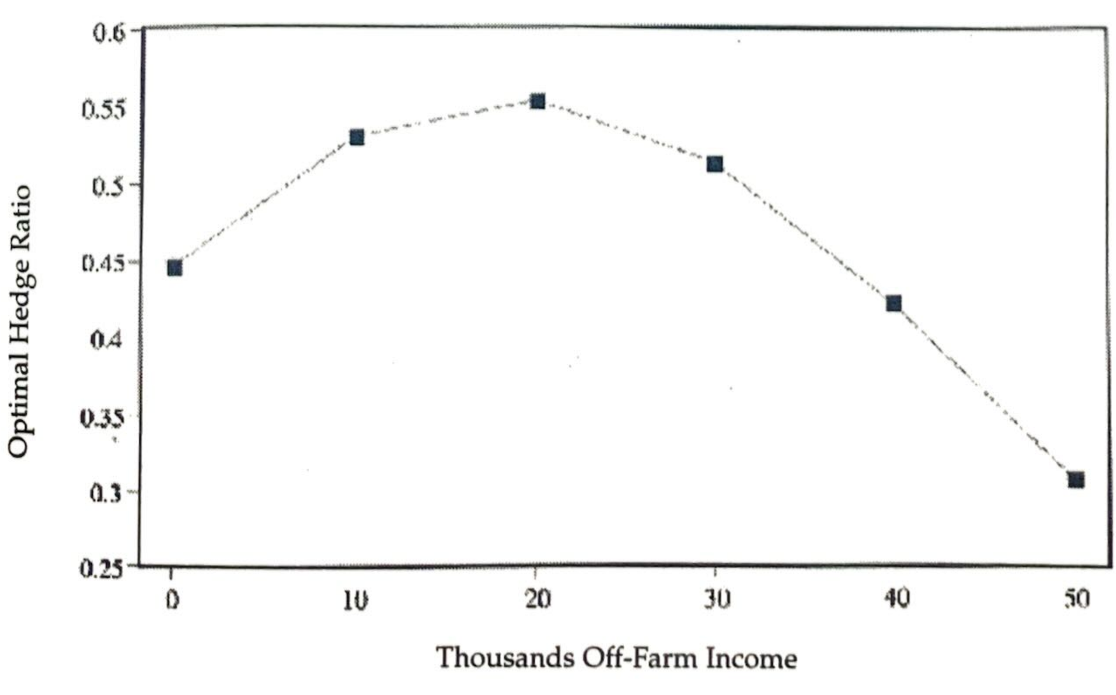


**Figure 6. Optimal hedge ratio for wheat vs. off-farm income ($thousands).**

nonlinear borrowing cost, progressive tax rates, and liquidity constraints

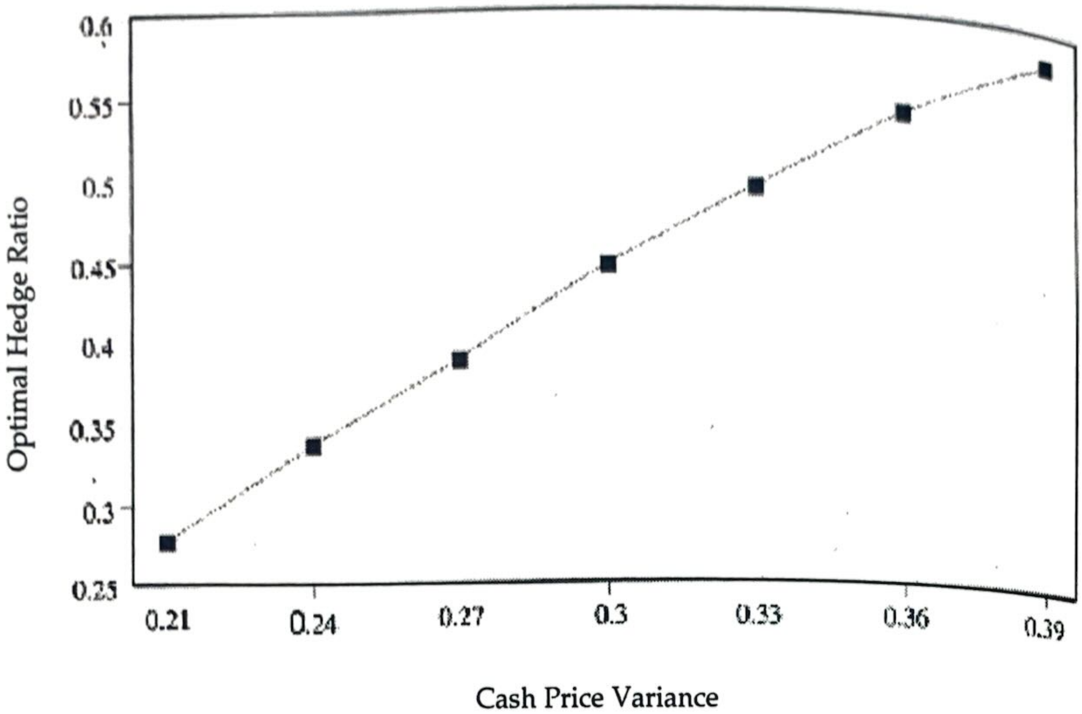


**Figure 7. Optimal hedge ratio for wheat vs. the cash price variance.**

## *Basis Risk*

As Castelino points out, as basis variance increases and holding other variables constant, farmers will have less incentive to hedge (fig. 8). A basis variance increase is modeled by reducing the correlation between futures and cash prices. Wheat optimal hedge ratios fall from a high of 0.45 to 0.26 when the correlation of cash and futures decreases from 0.9 to 0.75. For steers, further decrease in basis risk (from the base model, table 1) would not be enough incentive to hedge. The transaction cost of hedging is too high compared to the expected gain from hedging steers.

## *Deterministic Production*

When randomness of output was eliminated, the optimal hedge ratio for wheat increased from 0.45 to 0.48, a 6.7 % change. A recommended hedge ratio based on deterministic output can be suboptimal whenever the firm faces production uncertainty (as in most agricultural activities). As predicted by several authors

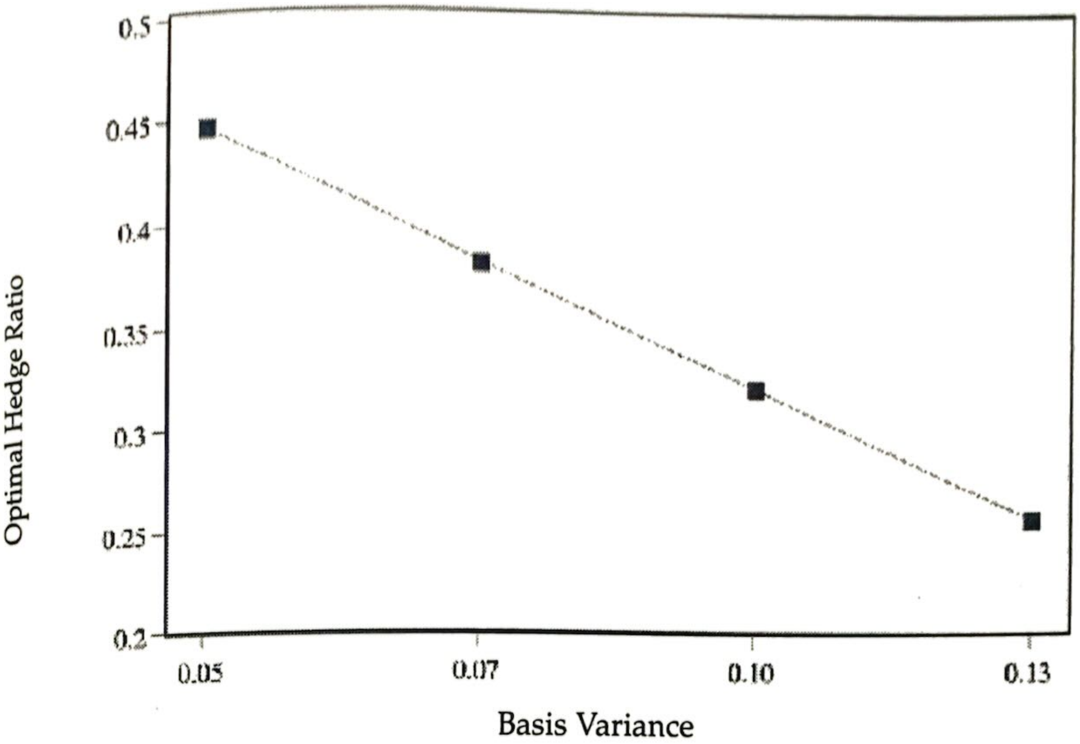


**Figure 8. Optimal hedge ratio for wheat vs. basis risk.**

(Chavas and Pope; Grant; Rolfo; Lence; Lapan and Moschini), production risk causes the optimal hedge ratio to be lower than in the case when output is nonrandom.

## *Progressive Tax Rates and Tax Carry Back*

In this model, taxes play a very important role in the decision to hedge, especially when the firm is not close to bankruptcy. A decrease in the marginal tax rate of 30% can make the optimal hedge ratio for wheat go to almost zero (fig. 9). On the other hand, an increase in the marginal tax rate of 30%, can bring the optimal hedge ratio to a high of 0.60. The tax reducing benefits of hedging provided the concavity of the objective function when the likelihood of positive bankruptcy and liquidity costs is zero. The agent need not be risk averse to hedge, instead, he/she hedges to reduce taxes and increase after-tax income.

With low profitability, the chances of having net operating losses (NOL) are high. Assuming all these losses could be carried back, the optimal hedge ratio would be zero (fig. 10). Even if the farmer was able to carry back only 50% of his/her NOL, the optimal hedge ratio would be less than 10%. Tax loss carry backs increase income in the current period and decrease the variability of income; therefore, reducing the incentive to hedge.

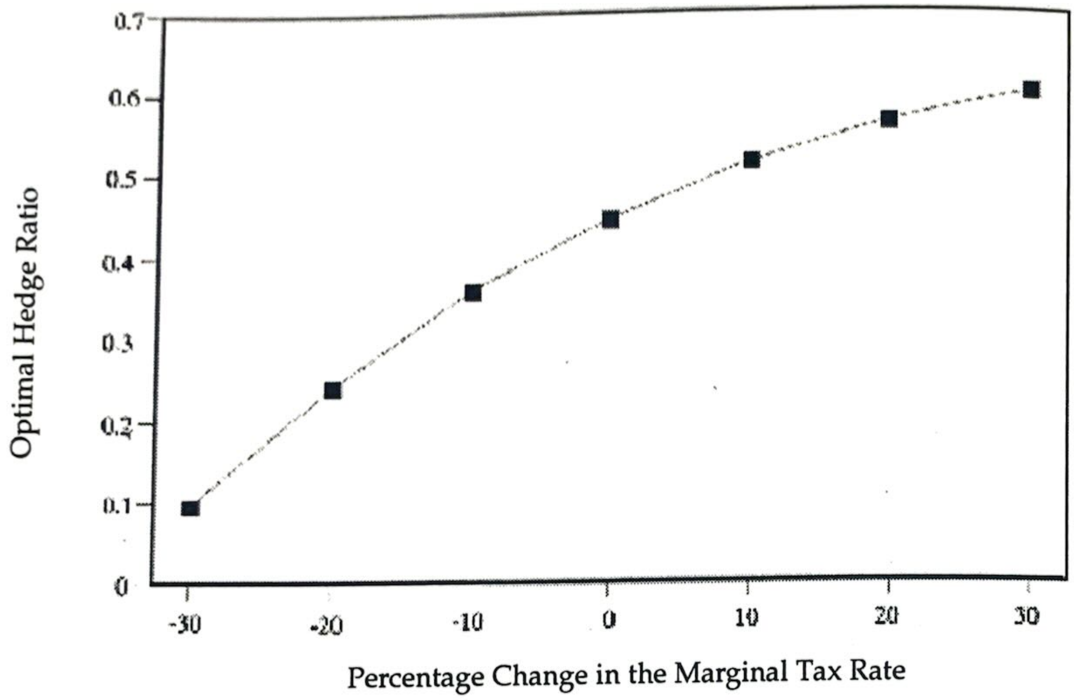


**Figure 9. Optimal hedge ratio vs. percentage change in the marginal tax rate.**

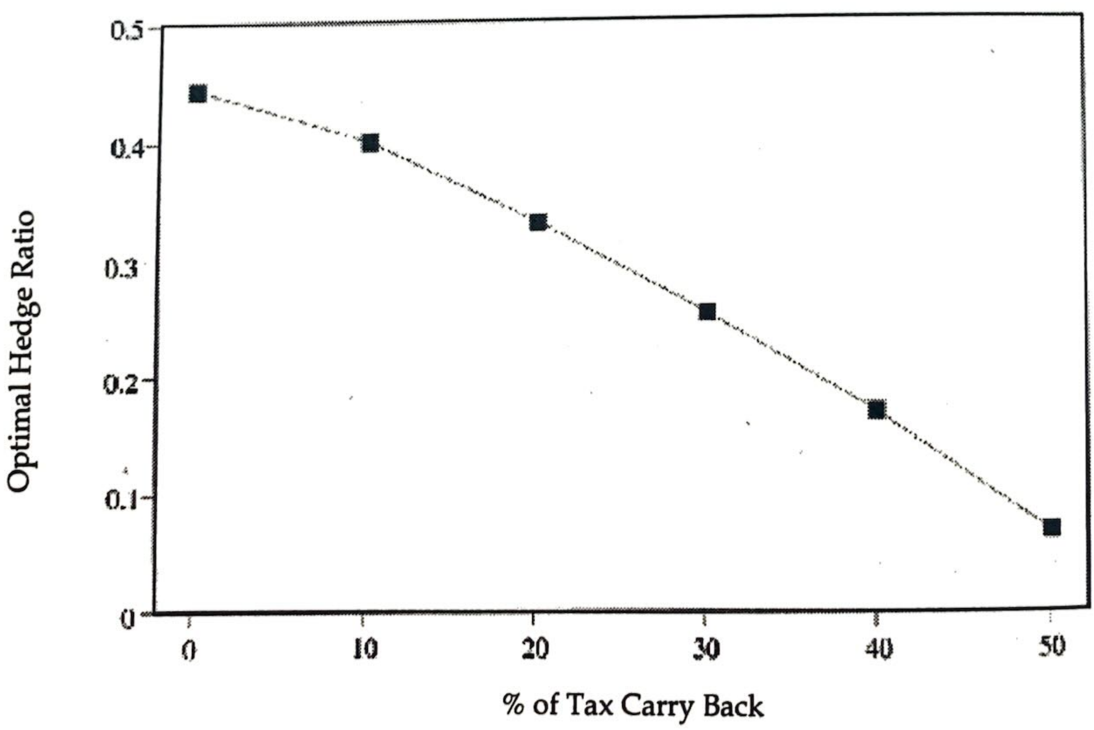


**Figure 10. Optimal hedge ratio vs. percentage of tax carry back.**

## CONCLUSIONS

Theoretical and empirical models usually suggest that farmers should hedge much more than they do. In this study, a new theoretical model of hedging is derived. In the model, the motivations to hedge are to reduce tax liabilities, bankruptcy costs, borrowing costs, and liquidity costs. Optimal levels of hedging are calculated for a hypothetical wheat and stocker steer producer. The model has much lower optimal hedge ratios than traditional models. The optimal hedge ratios reported here are very fragile depending on the costs of hedging. Empirical results show that a slight increase of the costs of hedging causes the optimal hedge ratios to greatly decrease or become zero. Optimal hedge ratios are quite sensitive to assumptions about costs because the benefits from hedging are small. Adding costs for the farmer's time or a farmer's dislike of futures could easily drive optimal hedge ratios to zero.

The results could explain why many firms do not hedge and some hedge more than others. The benefits and costs of hedging are small and thus individual differences can lead to very different optimal hedge ratios. The benefits of trading in the futures markets would be more significant if the marginal tax rates were higher. A flat tax rate with full loss carry back would eliminate the tax-reducing benefits of hedging.

When the probability of insolvency and bankruptcy is positive, the motivation to hedge is less tax related. The farmer is willing to bear the cost of hedging to reduce the relatively higher liquidity and bankruptcy costs. Hedging decreases the variability of profits, reducing expected liquidity and bankruptcy costs, and reducing interest.

A major implication of the results in this study is that risk-averse preferences are not necessary for farmers to hedge. Tax rates, liquidity costs and bankruptcy losses provide the concavity of the objective function necessary to motivate firms to hedge.

With empirical research usually suggesting farmers should hedge more than they do, some believe farmers should be taught the benefits of trading in the futures markets. The new theoretical model provides an explanation of why farmers would hedge so little. Extension economists should not treat every farmer the same and give them the same hedging recommendations.

Progressive tax rates introduce costs inefficiencies by motivating farmers to hedge more. A flatter tax schedule would reduce inefficiencies in the market. Futures exchanges should favor progressive tax rates since they lead to more hedging. Tax loss carry back, on the other hand, can eliminate the need for hedging when the firm experiences net operating losses. Similarly, for a high-income farm, income averaging would make farmers hedge less. It is possible that accounting tricks might balance income more cheaply than hedging. Also since farmers use cash accounting, if expected income for the year is high, farmers can reduce taxable income by buying inputs for next year in the current year.

Theoretical and empirical models used in past research have made simplifying assumptions that restrict them from explaining what farmers actually do. Future research on hedging needs to continue to seek models which can explain farmers' hedging behavior.

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